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A Game-Theoretic Price Discount Model in a Manufacturer-Retailer Channel

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Received: March 20, 2022 **Accepted:** June 18, 2022

Abstract: There are lots of price discount models, but game-theoretic models have not been used to model sequential price discount from the manufacturer to the consumer through the retailer. This work deals with a manufacturer-retailer price discount Stackelberg game model in which the manufacturer's discount can be given to the consumer through the retailer. The work models the players' payoffs using the players' price margins, discount rates and a linear demand function. Considering four scenarios the work obtains the players' optimal prices and payoffs. The work observes that each player performs better with non-provision of discount provided that the other channel member gives discount. Further, with equal discounts, the manufacturer's payoff is larger than the retailer's payoff for low discount rate. The reverse is the case for relatively large discounts. As the channel leader the manufacturer can thus opt for lower but equal discount rates. He can incentivise or where necessary constrain the retailer to give discount to the consumer without giving any to the retailer.

Keywords: Discount rate, linear demand function, price discount, price margin, manufacturer-retailer channel, Stackelberg game

Introduction

It is a known fact from the law of price and demand that price determines buyers' (retailers and consumers') interest in purchasing a product. Thus members of a supply chain are usually faced with the problem of appropriate price decisions. To woo potential buyers, channel members may provide price discount. This work is motivated by the scarcity of game theoretic models on price discount. Further, considering unhealthy rivalry that may result from blind uninformed approach in supply channels, this work will stand out to provide a platform for well informed decisions.

Chang *et al.* (2011) employed closed-form approach in solving Wee and Yu (1997) inventory model and Martin (1994) EOQ model with temporary price discount and sale price respectively. They proposed that a closed-form solution is achievable as against the two search methods (of Wee and Yu (1997) and Martin (1994)). In a study of an automobile supply chain involving subsidy and price discount Luo et *al.* (2014) observed that while subsidy ceiling is a more effective tool for a manufacturer with higher production cost, price discount is better option for a manufacturer with low production cost. They further observed that a discount rate and subsidy ceiling are necessary for effective incentive scheme. Paul *et al.* (2014) considered joint replenishment of multiple defective items using two scenarios: a model without price discount and a model with price discount. They observed that defectives affect the quantity to be ordered, order placement and total cost. Considering the design as well as the effectiveness of discount law remedy through comparative approach Mncube (2014) evaluated the claim that discount law remedy is beneficial. In an effort to study the effect of price discount on the perception of the consumer Lee and Chen-Yu (2018) developed a model, and that observed that consumers perceive products with high discount as low quality and those with low discount as high quality. Luo and Lee (2018) examined post-purchase discount negative effect, and suggested formats for discount promotion. A study of the effect of promotion, discount and social media on the purchase intention of consumers was carried out by Bhatti (2018). He observed that discount does not affect purchase intention. Using large empirical data from the market Choi and Chen (2019) considered how price discount and product bundling affect game-as-a-service applications. They observed that discount and bundling positively affect the applications' daily sales. Considering the evolution of the influence of the magnitude of price discount on the purchase intentions of the consumer, Sheehan *et al.* (2019) observed that providing low price discount at the beginning of an online shopping visit can be more effective for a firm. Still on online market, Wang *et al.* (2021) observed from a study on online book market that contrary to established economic predictions sellers' prices increase with increasing number of competitors. Thus, instead of reducing prices to give price discount, they rather increase their prices.

Game theory has been employed in price discount models. For instance, Sadjadi *et al.* (2018) used Stackelberg game to study interactions in a two-manufacturers-one retailer supply chain using price, price discount and provision of service. Tayor *et al.* (2019) combined game theory and Monte Carlo modelling to analyse different marketing strategies. They observed that this combination can be useful in making decisions. In general, game theoretic models have been very useful in studying the interactions in supply channels. An example can be found in Xie and Wei (2009). They considered a situation where the manufacturer's provided subsidy is given to the retailer. Other models on such interaction include Ezimadu and Ogini (20014), Ezimadu and Nwozo (2017), Ezimadu and Nwozo (2019).

In this work we employ the approach in these works to model a situation where the discount from the manufacturer is given to the retailer who in-turn can also provide discount to the consumer. Typically, we consider a price discount model in which the manufacturer gives back a fraction of the wholesale price to the retailer. The retailer in-turn gives a fraction of the retail price to the consumer.

Model Formulation

The Price-Demand Function

We examine a manufacturer-retailer channel in which the manufacturer who produces the goods, sells to the retailer who in-turn sells to the consumer. The retailer's decision variable is his retail price P_R while the manufacturer's decision variable is his wholesale price P_M. We assume that the law of price and demand for normal goods holds, and that the price-demand function is a linearly decreasing function of price (Jeuland and Shugan (1988), Weng (1995), Ezimadu (2019a, b). Thus we have

$$f(P_R) = 1 - \theta P_R \tag{1}$$

Where $\theta > 0$ is a constant representing the rate of change of demand with price?

We let λ and α denote the retailer and the manufacturer's discount rates respectively. They are fractions of the retail price and wholesale price which the retailer and manufacturer give to the consumer and retailer respectively.

The Payoffs

We note that

$$Payoff = Price\ Margin \times Demand - Expenditure. \tag{2}$$

We note that the retailer's price margin is $P_R - P_M$. Since the retailer receives a fraction of the manufacturer's price discount, and also gives a fraction of his price discount to the consumer we have that his expenditure can be expressed as $\lambda P_R - \alpha P_M$. Thus from the demand function (1) we have that the retailer's payoff is given by

$$R_{Pay} = (P_R - P_M)(1 - \theta P_R) - (\lambda P_R - \alpha P_M), \qquad (3)$$

Now, we note that the manufacture's wholesale price margin is $P_M - P_C$, where P_C is the production cost. The manufacturer gives discount as a fraction of his wholesale price margin to the retailer. This makes his expenditure to be αP_M. Thus, from the demand function (1) we have that the manufacturer's payoff is given by

$$M_{Pay} = (P_M - P_C)(1 - \theta P_R) - \alpha P_M. \qquad (4)$$

The Players' Problems, Strategies and Payoffs

Equilibrium Characterising Giving of Discounts by Both Manufacturer and Retailer

We use Stackelberg game to model this work, and consider the manufacturer as the channel leader, and the retailer as the follower. The manufacturer first announces his wholesale price P_M and price discount rate α. In reaction to this the retailer determines his retail price P_R and price discount rate λ. We will employ backward induction to obtain the Stackelberg equilibrium by first solving the retailer's problem

$$\max_{P_R>0} R_{Pay} = (P_R - P_M)(1 - \theta P_R) + \alpha P_M - \lambda P_R. \qquad (5)$$

From (5) we have

$$\frac{\partial R_{Pay}}{\partial P_R} = (P_R - P_M)(-\theta) + (1 - \theta P_R) - \lambda = 0,$$

Implying that

$$P_R = \frac{\theta P_M - \lambda + 1}{2\theta}. \qquad (6)$$

We note that the manufacturer's problem is given by

$$\max_{P_M>0} M_{Pay} = (P_M - P_C)(1 - \theta P_R) - \alpha P_M. \qquad (7)$$

Now, using (6) in (7) we have

$$\max_{P_M>0} M_{Pay} = (P_M - P_C)\left(\frac{\theta - \theta^2 P_M + \theta\lambda}{2\theta}\right) - \alpha P_M. \qquad (8)$$

Thus

$$\frac{\partial M_{Pay}}{\partial P_M} = (P_M - P_C)\left(-\frac{\theta^2}{2\theta}\right) + \left(\frac{\theta - \theta^2 P_M + \theta\lambda}{2\theta}\right) - \alpha = 0,$$

$$\Rightarrow \quad -\theta P_M + \theta P_C + 1 - \theta P_M + \lambda - 2\alpha = 0$$

$$\Rightarrow \quad P_M = \frac{\theta P_C + 1 + \lambda - 2\alpha}{2\theta}. \qquad (9)$$

Using (9) in (6) we have

$$P_R = \frac{\theta\left(\frac{\theta P_C + 1 + \lambda - 2\alpha}{2\theta}\right) - \lambda + 1}{2\theta}$$

$$= \frac{\theta P_C + 1 - \lambda - 2\alpha + 2}{4\theta}. \qquad (10)$$

Using (9) and (10) in (5)

$$R_{Pay} = \left(\frac{\theta P_C + 1 - \lambda - 2\alpha + 2}{4\theta} - \frac{\theta P_C + 1 + \lambda - 2\alpha}{2\theta}\right)\left(1 - \theta\frac{\theta P_C + 1 - \lambda - 2\alpha + 2}{4\theta}\right)$$

$$+\alpha\frac{\theta P_C + 1 + \lambda - 2\alpha}{2\theta} - \lambda\frac{\theta P_C + 1 - \lambda - 2\alpha + 2}{4\theta}$$

$$= \frac{1}{16\theta}\big((-\theta P_C + 1 - \lambda - 2\alpha)(-\theta P_C + 3 + \lambda + 2\alpha) + 8\alpha(\theta P_C + 1 + \lambda - 2\alpha)\big)$$

$$+\frac{1}{16\theta}\big(4\lambda(\theta P_C + 1 - \lambda - 2\alpha + 2)\big). \qquad (11)$$

Using (9) and (10) in (7) we have

$$M_{Pay} = \left(\frac{\theta P_C + 1 + \lambda - 2\alpha}{2\theta} - P_C\right)\left(1 - \theta\frac{\theta P_C + 1 - \lambda - 2\alpha + 2}{4\theta}\right) - \alpha\frac{\theta P_C + 1 + \lambda - 2\alpha}{2\theta}$$

$$= \frac{1}{8\theta}\big((1 + \lambda - 2\alpha - \theta P_C)(1 + \lambda + 2\alpha - \theta P_C) - 4\alpha(\theta P_C + 1 + \lambda - 2\alpha)\big). \qquad (12)$$

Thus:

Proposition 1 Suppose that both channel members give discount, then the retailer and manufacturer's price strategies are given by (10) and (9) respectively, and their payoffs are given by (11) and (12) respectively.

Equilibrium Characterising Non-Provision of Discounts by the Players

Since $\lambda = \alpha = 0$, (10) becomes

$$P_R = \frac{\theta P_C + 3}{4\theta}. \qquad (13)$$

Also (9) becomes

$$P_M = \frac{\theta P_C + 1}{2\theta}. \qquad (14)$$

Using (13) and (14) in (5) we have

$$R_{Pay} = \frac{1}{16\theta}(1 - \theta P_C)^2. \qquad (15)$$

Using (13) and (14) in (7) we have

$$M_{Pay} = \frac{1}{8\theta}(1 - \theta P_C)^2. \qquad (16)$$

Proposition 2 Suppose that neither of the channel members give discount, then the retailer and manufacturer's price strategies are given by (13) and (14) respectively, and their payoffs are given by (15) and (16) respectively.

Equilibrium Characterising the Manufacturer's Provision of Discount to the Retailer without the Retailer Extending the Discount to the Consumer

Based on this scenario (10) becomes

$$P_R = \frac{\theta P_C + 3 - 2\alpha}{4\theta}, \tag{17}$$

And (9) becomes

$$P_M = \frac{\theta P_C + 1 - 2\alpha}{2\theta}. \tag{18}$$

Using (17) and (18) in (5) we have

$$R_{Pay} = \left(\frac{\theta P_C + 3 - 2\alpha}{4\theta} - \frac{\theta P_C + 1 - 2\alpha}{2\theta}\right)\left(1 - \theta\frac{\theta P_C + 3 - 2\alpha}{4\theta}\right) + \alpha\frac{\theta P_C + 1 - 2\alpha}{2\theta}$$

$$= \frac{1}{16\theta}\left((1 + 2\alpha - \theta P_C)^2 + 8\alpha(\theta P_C + 1 - 2\alpha)\right). \tag{19}$$

Using (17) and (18) in (7) we have

$$M_{Pay} = \left(\frac{\theta P_C + 1 - 2\alpha}{2\theta} - P_C\right)\left(1 - \theta\frac{\theta P_C + 3 - 2\alpha}{4\theta}\right) - \alpha\frac{\theta P_C + 1 - 2\alpha}{2\theta}$$

$$= \frac{1}{8\theta}\left((1 - \theta P_C)^2 + 4\alpha^2 - 4\alpha(\theta P_C + 1)\right). \tag{20}$$

Thus

Proposition 3 Suppose that the manufacturer gives discount to the retailer, without the retailer giving discount to the consumer, then the retailer and manufacturer's price strategies are given by (17) and (18) respectively, and their payoffs are given by (19) and (20) respectively.

Equilibrium Characterising Retailer's Provision of Discount to the Consumer without Receiving Discount from the Manufacturer

Based on this scenario (10) becomes

$$P_R = \frac{\theta P_C + 3 - \lambda}{4\theta}, \tag{21}$$

and (9) becomes

$$P_M = \frac{\theta P_C + 1 + \lambda}{2\theta}. \tag{22}$$

Using (21) and (22) in (5) we have

$$R_{Pay} = \left(\frac{\theta P_C + 3 - \lambda}{4\theta} - \frac{\theta P_C + 1 + \lambda}{2\theta}\right)\left(1 - \theta\frac{\theta P_C + 3 - \lambda}{4\theta}\right) - \lambda\frac{\theta P_C + 3 - \lambda}{4\theta}$$

$$= \frac{1}{16\theta}\left((-\theta P_C - 3\lambda + 1)(-\theta P_C + \lambda + 1) - 4\lambda(\theta P_C - \lambda + 3)\right). \tag{23}$$

Using (21) and (22) in (7) we have

$$M_{Pay} = \left(\frac{\theta P_C + 1 + \lambda}{2\theta} - P_C\right)\left(1 - \theta\frac{\theta P_C + 3 - \lambda}{4\theta}\right)$$

$$= \frac{1}{8\theta}(1 - \theta P_C + \lambda)^2. \tag{24}$$

Thus:

Proposition 3-4 Suppose that the retailer gives discount to the consumer without receiving discount from the manufacturer, then the retailer and manufacturer's price strategies are given by (21) and (22) respectively, and their payoffs are given by (23) and (24) respectively.

Discussion

In this section we discuss the results obtained in this work. To aid the discussion we choose the following parameter values: Let $\alpha = 0.15$, $\lambda = 0.1$, $\theta = 0.2$ and $P_C = 0.5$. For simplicity we let the subscripts $\alpha = 0, \lambda = 0$; $\alpha \neq 0, \lambda = 0$; $\alpha = 0, \lambda \neq 0$ and $\alpha \neq 0, \lambda \neq 0$ represent a situation where neither the manufacturer nor the retailer gives discount; a situation where the manufacturer gives discount without the retailer extending same to the consumer; a situation where the retailer gives discount in the absence of discount from the manufacturer; and a situation where both provide discount.

Effect of Retailer's Discount on the Players' Payoffs

Considering Fig. 1 we observe that as the retailer's discount rate to the consumer increases, his payoff reduces, while the manufacturer's payoff increases. This is because the giving of discount can be seen as an expense incurred by the retailer which reduces his payoff.

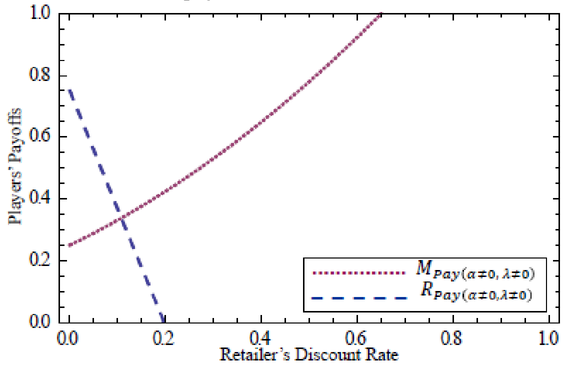

Fig. 1: Effect of retailer's discount rate on the players' payoffs

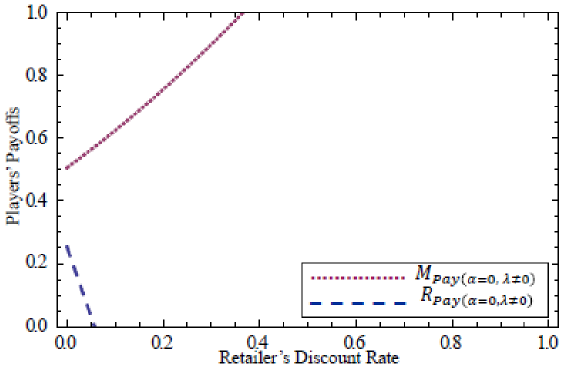

Fig. 2: Effect of retailer's discount rate on the players' payoffs when the manufacturer does not give discount.

A similar trend can be seen in Fig. 2 which shows that in the absence of any discount from the manufacturer, the retailer's payoff plunges more rapidly, while the manufacturer's payoff on the other hand increases more rapidly. Thus, the retailer should only provide discount when there is discount from the manufacturer. However, this should be done with restraint to only maintain the channel. Thus it is necessary to determine his optimal discount rate.

We note that the retailer's discount rate provides incentive for consumers' patronage of the product. The discount reduces his payoff when there is no discount inflow from the manufacturer. Since the manufacturer's payoff is based on patronage which increases the retailer's discount rate, his payoff tends to be unbounded with such high retail discount rate especially if he (the manufacturer) does not provide discount.

Effect of Manufacturer's Discount on the Players' Payoffs

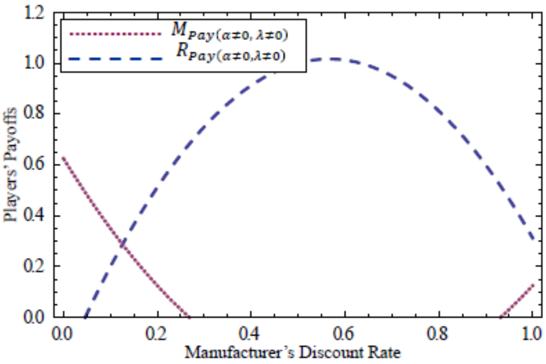

Fig. 3: Effect of manufacturer's discount rate on the players' payoffs

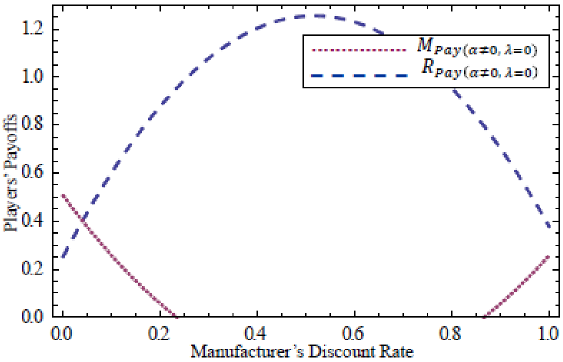

Fig. 4: Effect of manufacturer's discount rate on the players' payoffs when the retailer does not give discount to the consumer.

From Fig. 3 we observe a similar trend discussed above. Particularly, as the manufacturer's discount rate increases, his payoff reduces rapidly to zero while the retailer's payoff on the other hand increases, but eventually exhibits decrease. This decrease can be accounted for from equation (10) which indicates that the retail price reduces with increasing discount from the manufacturer. By extension, the marginal reduction affects the payoff. Thus a very high discount rate can be seen as not being too favourable to the retailer since it has the tendency of constraining him to reduce price. We observe a similar trend in Fig. 4 where retailer does not give discount. Since giving discount reduces his payoff rapidly, it is only rational for him to give discount when it is really necessary for the maintenance of the channel.

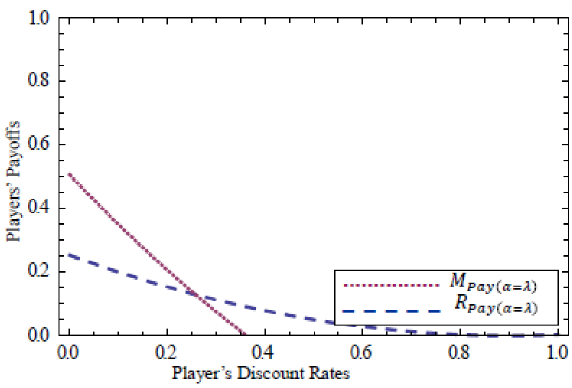

Fig. 5: Effect of equal discount rates on the players' payoffs.

From Fig. 1, Fig. 2, Fig. 3 and Fig. 4 it is obvious that it is necessary for the channel members to figure out a way of ensuring that they are not short-changed. Clearly Fig. 5 shows a situation where both discount rates are considered to be equal. It is obvious that with low discount rates the manufacturer's payoff is larger, but reduces with increasing discount rates making it eventually lower than the retailer's payoff. Further, we also observe that the manufacturer's payoff reduces more rapidly with discount. Since the manufacturer is the channel leader, he is positioned to control the flow of discount in the channel. It is only rational for him to give optimal discount, or simply give discount only up to the point where his payoff is larger than the retailer's payoff.

Comparison of Performances

Table 1: The players' optimal prices and payoffs for all scenarios

		Possible Scenarios			
		$\alpha = 0, \lambda = 0$	$\alpha \neq 0, \lambda = 0$	$\alpha = 0, \lambda \neq 0$	$\alpha \neq 0, \lambda \neq 0$
Prices and Payoffs	P_R	3.8750	3.5000	3.7500	3.3750
	P_M	2.7500	2.0000	3.0000	2.2500
	R_{Pa}	0.2531	0.7500	0.0000	0.5156
	M_{Pa}	0.5063	0.1500	0.6250	0.2313

Considering the scenario $\alpha \neq 0, \lambda = 0$, that is a situation where the manufacturer provides discount to the retailer, but the retailer does not give discount to the consumer, we observe that the retail price is relatively low compared to two other scenario, leading to the retailer's payoff being largest in this scenario. Thus the relatively low price $P_{R(\alpha\neq0,\lambda=0)}$ incentivises the consumer to patronise the product leading to large payoff. In essence, the discount from the manufacturer leads to lower retail price and consequently large payoff.

Similarly, we observe that for the scenario $\alpha = 0, \lambda \neq 0$, a situation where the retailer provides discount in the absence of discount from the manufacturer, the manufacturer's payoff is largest in this scenario. Thus irrespective of prices a particular player performs best in the discount rate scenario where the other player provides discount without that particular being involved.

Conclusion

This work considered a game theoretic discount provision model in a manufacturer-retailer channel in which the manufacturer is the Stackelberg leader while the retailer is the follower. This work is novel in the sense that it is the first game theoretic discount model on sequential transferable discount in a manufacturer-retailer setting. The work considered four discount scenarios, and obtained the players' optimal prices and payoffs for each scenario. We observe that each player performs better whenever they do not provide discount while the other player provides discount. Further, we observe that the manufacturer performs better than the retailer in a situation where the channel adopts equal discount rates for both channel members. The reverse is the case for large discount rates. That is, the retailer's payoff is larger than that of the manufacturer for relatively large discount rates. However, both payoffs are lower in this case. Thus, as the channel leader with first mover's advantage, the manufacturer can incentivise or where necessary constrain the retailer to give discount to the consumer while he holds back his discount. In the case of equal discounts, the manufacturer can opt for low discount since he is better-off with it.

This study has some limitations which can be explored to extend the work. This work used a linear demand function. An extension can employ a non-linear function. This can lead to a different result. Further, this work considered a manufacturer and a retailer channel. A consideration of multiple manufacturers and retailers can provide a better understanding of the effect of discount rates.

References

Bhatti A 2018. Sales promotion and price discount effect on consumer purchase intention with the moderating role of social media in Pakistan. *International Journal of Business Management*, 3(4): 50-58.

Chang H, Lin W & Ho J 2011. Closed-form solutions for Wee's and Martin's EOQ models with a temporary price discount. *International Journal of Production Economics*, 131(2): 528-534.

Choi HS & Chen, C 2019. The effects of discount pricing and bundling on the sales of game as a service: an empirical investigation, *Journal of Electronic Commerce Research*, 20(1): 21-34.

Ezimadu PE 2019a. A game-theoretic cooperative advertising model: the feasibility of the distributor's involvement in a manufacturer-distributor-retailer channel, *FUW Trends in Science & Technology Journal*, 4(2): 416-421.

Ezimadu PE 2019b. Modelling subsidy as a cooperative advertising channel coordination mechanism, *Nigerian Journal of Basic and Applied Science*, 27(2): 127-135.

Ezimadu PE & Nwozo CR 2017. Stochastic cooperative advertising in a manufacturer-retailer decentralized supply channel, *Journal of Industrial Engineering International*, 13: 1-12.

Ezimadu PE & Nwozo CR 2019. A manufacturer-retailers dynamic cooperative advertising with retail competition, *International Journal of Operational Research*, 34(1): 104-143.

Ezimadu PE & Ogini NO 2014. Modelling cooperative advertising, pricing and product quality in a manufacturer-retailer supply chain using game theory, *Journal of Mathematical Sciences*, 25(2): 69-83.

Jeuland AP & Shugan SM 1988. Channel of distribution profits when channel members form conjectures. *Marketing Science*, 7(2): 202–210.

Lee JE & Chen-Yu J. H. 2018. Effects of price discount on consumers' perceptions of savings, quality, and value for apparel products: mediating effect of price discount affect, *Fashion and Textiles*, 5:13 https://doi.org/10.1186/s40691-018-0128-2

Luo C, Leng M, Huang J & Liang L 2014, Supply chain analysis under a price-discount incentive scheme for electric vehicles. *European Journal of Operational Research*, 235(1):329-333.

Luo X & Lee JJ 2018. The Effect of Post-Purchase Discount Format on Consumers' Perception of Loss and Willingness to Return, *Journal of Asian Finance, Economics and Business*, 5(4): 101-105.

Martin GE 1994. Note on an EOQ model with a temporary sale price. *International Journal of Production Economics*, 37(2): 241-243.

Mncube L 2014. Settling for a discount: a review of the pioneer foods price reduction remedy, *Agrekon: Agricultural Economics Research, Policy and Practice in Southern Africa*, 53(1): 26-45.

Paul S, Wahab MIM, Ongkunaruk P 2014. Joint replenishment with imperfect items and price discount. *Computers & Industrial Engineering*, 74(1): 179-185.

Sheehan D, Hardesty DM, Ziegler AH and Chen H 2019. Consumer reactions to price discounts across online shopping experiences, *Journal of Retailing and Consumer Services*, 51 :129–138.

Sadjadi SJ, Asadi H, Sadeghian R & Sahebi H 2018. Retailer Stackelberg game in a supply chain with pricing and service decisions and simple price discount contract, *PLoS ONE* 13(4): e0195109. https://doi.org/10.1371/journal. pone.0195109

Taylor M, Kwasnica V, Reilly D & Ravindran S 2019. Game theory modelling of retail marketing discount strategies. *Marketing Intelligence and Planning*, 37(5): 555-566.

Wang W, Li F & Zhang Y 2021. Price discount and price dispersion in online market: do more firms still lead to more competition? *Journal of Theoretical and Applied Electronic Commerce Research*, 16(2):140-154.

Wee HM & Yu J 1997. A deteriorating inventory model with a temporary price discount. *International Journal of Production Economics*, 53(1): 81-90.

Weng ZK 1995. Channel coordination and quantity discounts. *Management Science*, 41(9): 1509-1521.

Xie J & Wei J 2009. Coordination advertising and pricing in a manufacturer-retailer channel. *European Journal of Operational Research*, 197(2): 785-791.